
📶 Example: 4-QPSK OFDM System with Cyclic Prefix and Channel

This walks through the full process:

1. QPSK symbols
2. IFFT to generate OFDM symbol
3. Cyclic Prefix insertion
4. Channel convolution
5. Cyclic Prefix removal
6. FFT at receiver
7. Equalization

Step-by-Step OFDM Example

🔣 Step 1: QPSK Data Symbols

4 QPSK symbols (on 4 subcarriers):

$X=\{1+i, -1+i, -1-i, 1-i\}$

🔁 Step 2: IFFT → Time-domain OFDM symbol

Take a 4-point IFFT:

$x=IFFT(X)$

This gives us:

$x=\{0, 1-i, 0, 1+i\}$

🧱 Step 3: Add Cyclic Prefix

Use CP length = 2 (copy last 2 samples to the beginning):

$xcp=\{0,1+i, 0, 1-I ,0,1+i\}$

📡 Step 4: Pass Through Channel

Use a 2-tap channel:

h={1,0.5}

Apply linear convolution:

y=xcp∗h

Example result (rounded):

y={0,1+i, 0.5+0.5i, 1+0i, 0.5+1.5i, 0.5+0.5i, 0.5+0.5i

🧠 Step 5: Remove Cyclic Prefix

Remove first 2 samples:

yrx={0.5+0.5i, 1+0i, 0.5+1.5i, 0.5+0.5i}

⚡ Step 6: FFT at Receiver

Apply FFT to `y_rx`:

Y=FFT(yrx)

Result (sample):

Y={1.5+2.5i,0.5−1.5i,−1.5+0.5i,0.5+0.5i}

🎯 Step 7: Equalize Assuming Channel response is known

Get FFT of the channel (padded to 4):

H=FFT({1, 0.5, 0, 0})={1.5, 1+0.5i, 0.5, 1−0.5i}

Now divide element-wise:

Xdet=Y/H = Y / H

Recovered QPSK symbols (approx):

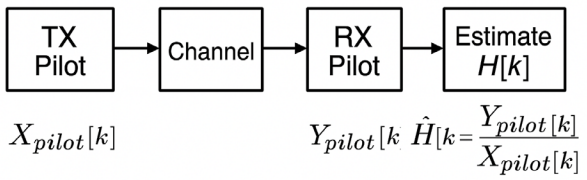
Xdet={1+i, −1+i, −1−i, 1−i}

✅ Summary Table

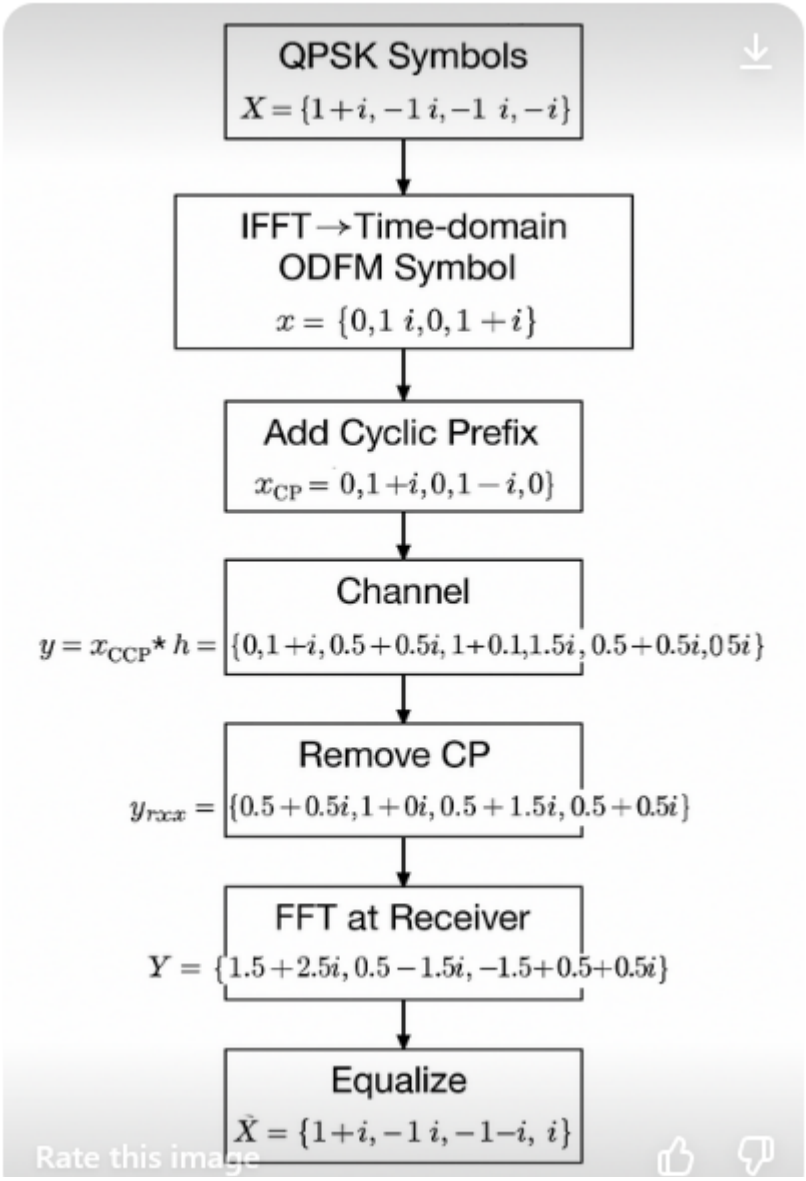
Stage	Result
QPSK symbols	{1 + i, -1 + i, -1 - i, 1 - i}
IFFT	{0, 1 - i, 0, 1 + i}
With CP	{0, 1 + i, 0, 1 - i, 0, 1 + i} Use a 2-tap channel h={1,0.5}
Channel Output	{0.5 + 0.5i, ..., 0.5 + 0.5i}
After CP Removal	{0.5 + 0.5i, 1, 0.5 + 1.5i, 0.5 + 0.5i}
FFT (Y[k])	{1.5 + 2.5i, ..., 0.5 + 0.5i}
Channel FFT (H[k])	{1.5, 1 + 0.5i, 0.5, 1 - 0.5i} Assuming H known
Equalized Symbols	{1 + i, -1 + i, -1 - i, 1 - i}

In **real-world OFDM systems**, the channel impulse response h[n] is **unknown**.

H{k] is e**stimated H[k]H[k]** using **pilot symbols** or **training sequences** that are known to both the transmitter and receiver.



From: ChatGPT version GPT-4-turbo


QPSK Symbols
$X = \{1+i, -1\,i, -1\;\; i, -i\}$
IFFT → Time-domain ODFM Symbol
$x = \{0, 1\;i, 0, 1+i\}$
Add Cyclic Prefix
$x_{CP} = 0, 1+i, 0, 1-i, 0\}$
Channel
$y = x_{CCP} * h = \{0, 1+i, 0.5+0.5i, 1+0.1, 1.5i, 0.5+0.5i, 0\,5i\}$
Remove CP
$y_{rxx} = \{0.5+0.5i, 1+0i, 0.5+1.5i, 0.5+0.5i\}$
FFT at Receiver
$Y = \{1.5+2.5i, 0.5-1.5i, -1.5+0.5+0.5i\}$
Equalize
$\hat{X} = \{1+i, -1\,i, -1-i, \;i\}$
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